<u>Climate Sensitivity & feed back application</u>

To gain a more quantitative understanding of the climate response to a prescribed forcing, it is useful to place the concepts of climate feedbacks and climate sensitivity into a mathematical frame work.

The radiative forcing F is defined as the net downward flux density or irradiance at the top of the atmosphere that would result if it were applied instantaneously.

For example, if a climate variable like global mean surface air temperature T_s depends on three factors. They are incident solar radiation, the planetary albedo and the strength of the green house effect averaged over the Earth's surface. A number of auxiliary variables like the atmospheric water vapor, ozone, clouds, ice and snow also are to be taken into consideration while determining the albedo and the strength of the green house effect.

Let suppose, the surface temperature T_s is increased by dT_s due to the increase of downward flux density of solar radiation 'ds'. Then the sensitivity of T_s to the radiative forcing F, is given as $\lambda = \frac{dT_s}{dF}$ which is called the climate sensitivity. This can be further written as

$$\lambda = \frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \sum_i \frac{\partial T_s}{\partial y_i}\frac{dy_i}{dF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where y_i is an auxiliary variable like the atmospheric water vapor, ice, low clouds etc. which influence T_s.

The first term of equation (1) is $\lambda_0 = \frac{\partial T_s}{\partial F}$ is the climate sensitivity that would prevail in the absence of feed backs involving the auxiliary variables and can be written as

$$\lambda_0 = \frac{\partial T_s}{\partial F} = \frac{dT_E}{dF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Where T_E is the equivalent black body temperature of earth.

<u>Question 1:</u>

Calculate the equivalent black body temperature of the earth if its planetary albedo is 0.30.

Ans: Let F_s be the flux density of solar radiation incident upon the earth = 1368 wm^{-2} and let the flux density of long wave radiation emitted by the earth is F_E. Then

$$F_E = \frac{\pi R^2 F_s}{4\pi R^2}(balance of albedo) \Rightarrow F_E = \frac{(1-A)F_s}{4} = \frac{(1-0.30)1368}{4} = 239.4\,\text{wm}^{-2} \ldots\ldots..(3)$$

Where A = 0.30 is the albedo and R is the radius of the earth.

According to Stefan- Boltzman law $F_E = \sigma T_E^{\,4}$ or $T_E = \left[\frac{F_E}{\sigma}\right]^{1/4}$ $\ldots\ldots\ldots(4)$

$$\therefore T_E = \left[\frac{239.4}{5.67\times 10^{-8}}\right]^{1/4} = 255K \quad \ldots\ldots\ldots\ldots\ldots(5)$$

<u>Question 2:</u>

Estimate the sensitivity of the earth's equivalent black body temperature to a change in the solar radiation F_E incident upon the top of the atmosphere.

Ans: From equation (4) we know $T_E = \left[\frac{F_E}{\sigma}\right]^{1/4}$ Taking the natural logs on both sides we can write $\ln T_E = \frac{1}{4}\ln F_E - \frac{1}{4}\ln\sigma$ Differentiating this eqn. we get

$\frac{1}{T_E}dT_E = \frac{1}{4}\frac{1}{F_E}dF_E - 0$ (as σ is Stefan Boltzman constant)

We can further write $\frac{dT_E}{dF_E} = \frac{1}{4}\frac{T_E}{F_E}$ ………………….(5)

From eqn (3) we know F_E for the earth as 239.4 and from eqn (5) we know T_E for the earth as 255K .Substituting these values in eqn.(5) we get

$$\frac{dT_E}{dF_E} = \frac{1}{4}\frac{T_E}{F_E} = \frac{1}{4}\left[\frac{255}{239.4}\right] = 0.266\,K(wm^{-2})^{-1} \quad \text{………..(6)}$$

Or taking the reciprocal , it can be inferred that the Earth's equivalent black body temperature increases by 1 k for every 3.76 wm^{-2} of downward radiative forcing at the top of the atmosphere.

$$\therefore Radiativeforcing = \frac{1}{0.266} = 3.76(wm^{-2}/1^0 K) \quad \text{…………………(7)}$$

Auxiliary variable of equation 1:

The change in the auxiliary variable in the last term of equation (1) can be written as

$$\frac{dy_i}{dF} = \frac{dy_i}{dT_S}\frac{dT_S}{dF} \quad \text{……………………………….(8)}$$

Substituting equation (7) in equation (1) we can write

$$\frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \sum_i \frac{\partial T_s}{\partial y_i}\frac{dy_i}{dT_S}\frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \frac{dT_s}{dF}\sum_i \frac{\partial T_s}{\partial y_i}\frac{dy_i}{dT_S} = \frac{\partial T_s}{\partial F} + \frac{dT_s}{dF}\sum_i f_i \quad \text{……..(9)}$$

Where $\sum_i \frac{dy_i}{dT_S}\frac{\partial T_s}{\partial y_i} = \sum_i f_i = f(say)$ ……………………….(10)

The term f_i is a feed back factor associated with various feed back processes like water vapor, cloud forcing, ice albedo, CO_2 etc.

The feed back factor 'f_i' will be positive if the two terms in equation (9) have same sign. It will be negative if they have opposing sign.

Equation (8) can further be written as

$$\frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \frac{dT_s}{dF}f_i$$

$$\frac{dT_s}{dF}(1-f) = \frac{\partial T_s}{\partial F}$$

$$or\,\frac{dT_s}{dF} = \frac{\partial T_s}{\partial F}\Big/(1-f) \quad \text{…………………….(11)}$$

Hence the gain $g = \lambda / \lambda_0$ due to the presence of climate feed back is

$$g = \frac{1}{1-f} \quad \text{.............................(12)}$$

i) 'g' is positive if f is less than unity ($f < 1$).

ii) If $f \geq 1$, g is negative or infinity which means sensitivity is infinite. This means even an infinitesimal forcing may cause the climate system to diverge from its present equilibrium state and seek a new one.

Question 3:

a) Estimate the apparent climate sensitivity $\left(\frac{\delta T_S}{\delta F}\right)$ between the present climate and the climate at the time of last glacial maximum (LGM) around 20, 000 years ago taking the following factors into consideration:

i) The global air temperature at LGM is 5^0C less than today

ii) The climate forcing due to doubling of CO_2 concentration is 3.7 w m^{-2} at present according to radiative transfer model calculation

iii) The radiative forcing due to planetary albedo was 0.01times higher in LGM than today

iv) Assume the flux density of solar radiation at the time of LGM and today are same (342 w m^{-2}) as there is not much change in solar cycle.

Ans: $$\left(\frac{\delta T_S}{\delta F}\right) = \frac{T_S(present) - T_S(LGM)}{F(present) - F(LGM)}$$

The difference of temperature is given as 5^0C = 5^0K = (T_s present - T_s LGM)

The flux of reflected radiation at the top of the atmosphere at LGM is = $342 \times 0.01 = 3.42$ w m^{-2}

Substituting these values in the above equation we get

$$\left(\frac{\delta T_S}{\delta F}\right) = \frac{5K}{\{3.7-(-3.42\}wm^{-2}} = 0.70 \text{ K per w m}^{-2} \quad \text{.......................(13)}$$

b) Find the influence of feed backs by comparing the result of question (2) (equation 7) with the climate variability in the absence of feed backs (λ_0) and the apparent sensitivity of the climate system between present and past climates of LGM

Ans:

The result of question (2) (equation 6) is $\frac{dT_E}{dF_E} = \frac{1}{4}\frac{T_E}{F_E} = \frac{1}{4}\left[\frac{255}{239.4}\right] = 0.266\,K(wm^{-2})^{-1}$

From equation (13) the apparent climate sensitivity in the presence of feed back is

$\left(\frac{\delta T_S}{\delta F}\right)$ =0.70 K per w m^{-2}

The enhanced factor between the two climates due to the presence of feed backs $= \frac{0.70}{0.266} = 2.7$

This means due to the presence of feed backs the climate system is enhanced by a factor = 2.7